SELETIVA - LISTA 01

Matheus Ponciano

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1. Bicicletou

A wheel with spokes rolls without slipping on the ground. A stationary camera takes a picture of it as it rolls by, from the side. Due to the nonzero exposure time of the camera, the spokes generally appear blurred. At what locations in the picture do the spokes not appear blurred? Hint: A common incorrect answer is that there is only one point.

2. Porta giratória

A uniform rectangular door of mass with sides and and negligible thickness rotates with constant angular velocity about a di agonal. Ignore gravity. Find the torque that must be applied to keep the door rotating.

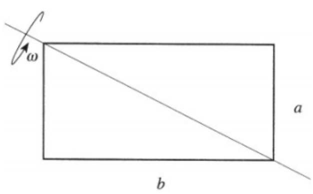


Figura 1: Problema 2

3. Baratinha do Aldeota

A toy globe rotates freely without friction with an initial angular velocity ω_o A bug starting at one pole N travels to the other pole S along a meridian with constant velocity v The axis of rotation of the globe is held fixed. Let M and R denote the mass and radius of the globe (a solid sphere, moment of inertia $I_o = \frac{2}{5}MR^2$ the mass of the bug, and T the duration of the bug's journey.

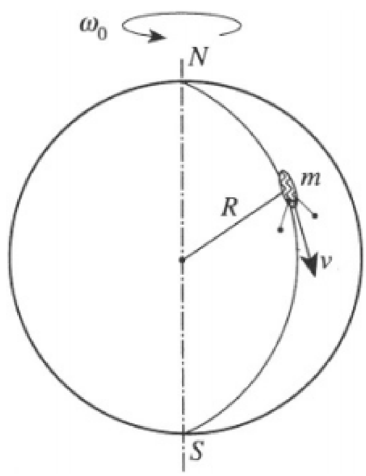


Figura 2: Problema 3

Show that, during the time the bug is traveling, the globe rotates through an angle

$$\Delta\theta = \frac{\pi\omega_o R}{v}\sqrt{\frac{2M}{2M+5m}}$$

A useful integral is

$$\int_0^{2\pi} \frac{dx}{a+bcos(x)} = \frac{2\pi}{\sqrt{a^2-b^2}}$$

4. Axle Rose

A massless axle has one end attached to a wheel (a uniform disk of mass m and radius r), with the other end pivoted on the ground. The wheel rolls on the ground without slipping, with the axle inclined at an angle θ. The point of contact on the ground traces out a circle with frequency Ω.

(a) Show that ω points horizontally to the right (at the instant shown), with magnitude $\omega = \Omega/tan\theta$.

(b) Show that the normal force between the ground and the wheel is

$$N = mgcos^2\theta + mr\Omega^2\left(\frac{1}{4}cos\theta sen^2\theta + \frac{3}{2}cos^3\theta\right)$$

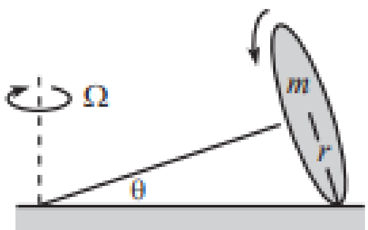


Figura 3: Problema 4

5. Tadeu Momento

If the line of action of the impulse in the previous problem does not lie in the vertical plane defined by the points T, C and P, then, just after the shot, the ball's angular velocity vector will not be perpendicular to the velocity of its centre of mass. Billiards players call this shot a Coriolis-massé. Such a shot is shown in the figure, in which the line of action of the impulse meets the ball's surface (for a second time) at T' and the table at A.

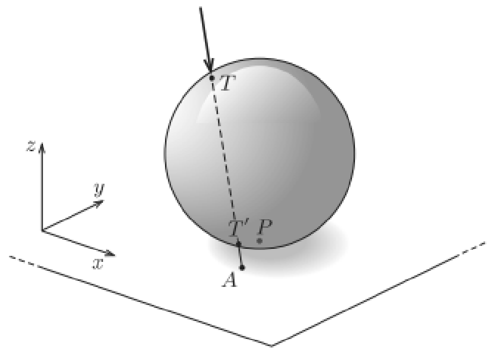


Figura 4: Problema 5

(a) What kind of trajectory does the ball's centre of mass follow from just after the shot until the point at which simultaneous rolling and slipping cease?

(b) In which direction, relative to the line PA, will the ball continue its path once it starts to roll without slipping? Assume that, whatever the downward force acting on it, the billiard cloth does not 'become squashed', and the ball's contact with it is always a point contact.

6. Truque de Mágica

A large flat disc with a rough surface rotates around the axis of symmetry that is normal to its plane, and does so with constant angular velocity Ω. The plane of the disc is tilted at an angle θ relative to the horizontal.

A magician places a solid rubber ball of radius R and mass m on the rotating disc and starts it off in an appropriate direction. Then, to the audience's great surprise, the centre of the ball moves uniformly in a straight line, until it reaches the rim of the rotating disc. Throughout the ball's motion, it does not slip on the disc, and the angular velocity of the disc does not change.

Find a physical explanation for this strange phenomenon. In which direction, and how quickly, should the magician start the ball for this stunt to be successful?

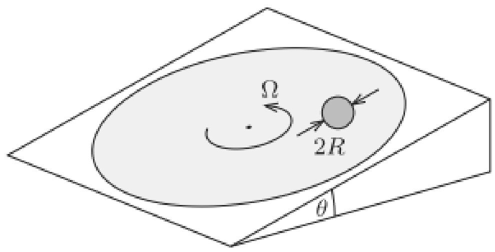


Figura 5: Problema 6

7. Rolando estranho

The standard way that a ball rolls without slipping on a flat surface is for the contact points on the ball to trace out a vertical great circle on the ball. Are there any other ways that a ball can roll without slipping?

8. Rolando estranho e reto?

In some situations, such as the rolling-coin setup in Problem 7, the velocity of the CM of a rolling object changes direction as time goes by. Consider a uniform sphere that rolls on the ground without slipping (possibly in the nonstandard way described in the solution to Problem 7). Is it possible for the CM's velocity to change direction?

9. Terremoto

A uniform ball rolls without slipping on a table (possibly in the nonstandard way described in the solution to Problem 7). It rolls onto a piece of paper, which you then slide around in an arbitrary (horizontal) manner. You may even give the paper abrupt, jerky motions, so that the ball slips with respect to it. After you allow the ball to come off the paper, it will eventually resume rolling without slipping on the table. Show that the final velocity equals the initial velocity.

10. Agora Complicou

A uniform ball rolls without slipping on a turntable (possibly in the nonstandard way described in the solution to Problem 7). As viewed from the inertial lab frame, show that the ball moves in a circle (not necessarily centered at the center of the turntable) with a frequency equal to 2/7 times the frequency of the turntable.

11. Chapéu de Mago

A ball (with $I = \frac{2}{5}MR^2$) rolls without slipping on the inside surface of a fixed cone, whose tip points downward. The half-angle at the vertex of the cone is θ. Initial conditions have been set up so that the contact point on the cone traces out a horizontal circle of radius $l >> R$, at frequency Ω, while the contact point on the ball traces out a circle of radius r (not necessarily equal to R, as would be the case for a great circle). Assume that the coefficient of friction between the ball and the cone is sufficiently large to prevent slipping. What is the frequency of precession, Ω? It turns out that Ω can be made infinite if $\frac{r}{R}$ takes on a particular value; what is this value? Work in the approximation where $R << l$.

12. Parádoxo dos Golfinhos

A small solid rubber ball of radius r is thrown onto the inner wall of a long cylindrical tube, which has radius R and is fixed with its axis of symmetry vertical. If the ball is started off with a sufficiently large horizontal velocity v_o, then it starts to oscillate periodically in the vertical direction, while still maintaining contact with the tube. Describe the 'dance' performed by the centre of the ball. The static friction is quite large, and so the ball never slides on the wall. Assume that the ball is sufficiently incompressible that its contact with the tube is only ever through a single point, and that air drag and rolling friction are negligible.

13. NUTação

Assume that uniform circular precession is initially taking place with $\theta = \theta_o$ and $\Phi = \Omega_s$. You then give the top a quick kick along the direction of motion, so that Φ suddenly becomes $\Omega_s + \Delta\Omega$ ($\Delta\Omega$ may be positive or negative). Find $\Phi(t)$ and $\theta(t)$.

Assume that the top is spinning rapidly $\left(\omega_3 >> \Omega_s = \frac{Mgl}{I_3\omega_3}\right)$ and that $sen\theta \approx sen\theta_o$.

Dica: Lembre do torque no heavy top e de nutação.

As próximas duas questões são de provas da Apho e da Ipho. Se você não quiser fazê-las para evitar estragar simulado. A última página é para fontes e referências, então é seguro pular para ela.

14. Foguetas

Introduction

In more than half a century of space operations quite a large number of man-made objects have been amassed near Earth. The objects that do not serve any particular purpose are called space debris. The most attention is usually paid to the larger debris objects, i.e. defunct satellites and spent rocket upper stages, which stay in orbit after delivering their payload. Collisions of such objects with each other may result in thousands of fragments endangering all current space missions.

There is a well-known hypothetical scenario, according to which certain collisions may cause a cascade where each subsequent collision generates more space debris that increase the likelihood of new collisions. Such a chain reaction, resulting in the loss of all near-Earth satellites and making impossible further space programs, is called the Kessler syndrome.

To prevent such undesirable outcome special missions are planned to remove large debris object from their present orbits either by tugging them to the Earth's atmosphere or to graveyard orbits. To this end a specially designed spacecraft – a space tug – must capture a debris object. However, before capturing an uncontrolled object it is important to understand its rotational dynamics. We suggest you to take part in planning of such a mission and find out how the rotational dynamics of a debris object changes in time under the influence of different factors.

Rocket Stage Schematic

The debris object to be considered is a "Kerbodyne 42"rocket upper stage, whose schematic is shown in Fig. 1. The circle line in Fig. 1 marks the outline of a spherical fuel tank.

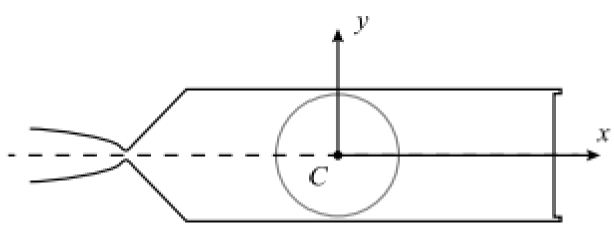


Fig. 1: "Kerbodyne 42" upper stage

We introduce a body-fixed reference frame C_{xy} with the origin in the center of mass C, x being the symmetry axis of the stage, and y perpendicular to x. The inertia moments with respect to x and y axes are J_x and J_y ($J_x < J_y$).

Part A. Rotation (3.8 points).

Consider an arbitrary initial rotation of the stage with angular momentum L (Fig. 2), where θ is the angle between the symmetry axis and the direction of angular momentum. Fuel tank at this point is assumed to be empty. No forces or torques act upon the stage.

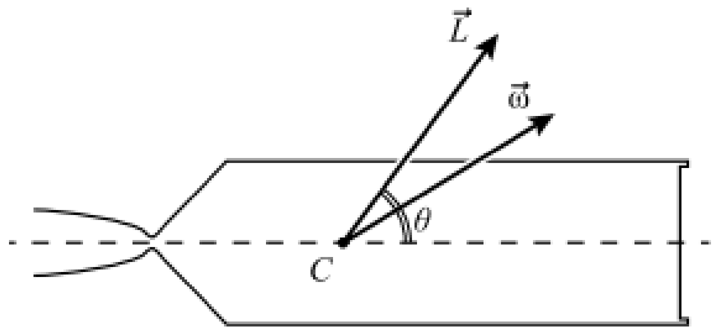


Fig. 2: Rocket stage rotation

A.1 Find the projections of angular velocity $\vec{\omega}$ on x and y, given that $\vec{L} = J_x\vec{\omega}_x\hat{e}_x + J_y\omega_y\hat{e}_y$ for material symmetry axes x and y with unit vectors $\hat{e}_x$ and $\hat{e}_y$. Provide the answer in terms of $L = |\vec{L}|$, angle θ, and inertia moments J_x , J_y.

A.2 Find the rotational energy E_x associated with rotation ω_x and E_y associated with rotation ω_y. Find total rotational kinetic energy $E = E_x + E_y$ of the stage as a function of the angular momentum L and $cos\theta$.

In the following questions of Section A consider the stage's free rotation with the initial angular momentum L and $\theta(0) = \theta_o$.

A.3 Let us denote by x_o the initial orientation of the stage's symmetry axis C_x with respect to inertial reference frame. Using conservation laws find the maximum angle ψ, which the stage's symmetry axis C_x makes with x_o during the stage's free rotation.

Note: Since there are no external torques acting upon the stage, the angular momentum vector remains constant.

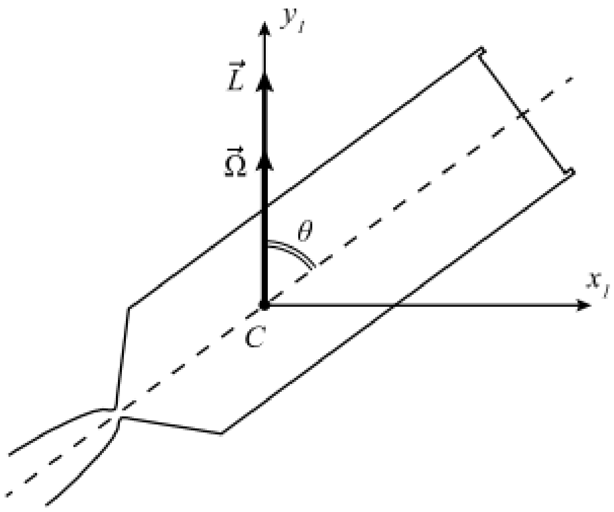


Fig. 3: Precession

Let us now introduce the reference frame $C_{x_1y_1z_1}$ with y_1 along the constant angular momentum vector $\vec{L}$ (Fig. 3). This reference frame rotates about y_1 in such a way, that the stage's symmetry axis always belongs to the $C_{x_1y_1}$ plane.

A.4 Given L, $\theta(0) = \theta_o$, and inertia moments J_x , J_y , find the angular velocity $\Omega(t)$ of the reference frame $C_{x_1y_1}$ about y_1 and direction and absolute value of angular velocity of the stage $\vec{\omega}_s(t)$ relative to the reference frame $C_{x_1y_1}$

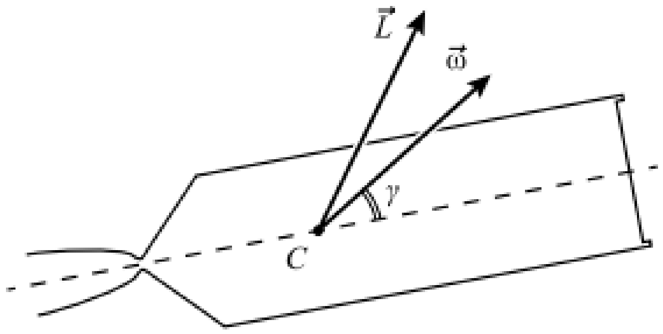
as functions of time. Provide the answer for $\vec{\omega}_s(t)$ direction in terms of angle $\gamma_s(t)$ it makes with the symmetry axis C_x.

Note: angular velocity vectors are additive $\vec{\omega} = \vec{\omega}_x + \vec{\omega}_y = \vec{\Omega} + \vec{\omega}_s$

Part B. Transient Process

Most of the propellant is used during the ascent, however, after the payload has been separated from the stage, there still remains some fuel in its tank. Mass m of residual fuel is negligible in comparison to the stage's mass M. Sloshing of the liquid fuel and viscous friction forces in the fuel tank result in energy losses, and after a transient process of irregular dynamics the energy reaches its minimum.

B.1 Find the value θ_2 of angle θ after the transient process for arbitrary initial values of L and $\theta(0) = \theta_1 \in (0, \pi/2)$.



Angle between the stage's angular velocity and the symmetry axis

B.2 Calculate the value ω_2 of angular velocity ω after the transient process, given that initial angular velocity $\omega(0) = \omega_1 = 1\ rad/s$ makes an angle of $\gamma(0) = \gamma_1 = 30^o$ with the stage's symmetry axis. The moments of inertia are $J_x = 4200\ kg.m^2$ and $J_y = 15000\ kg.m^2$.

15. Halteres Interes

A weightless rod of a length $2R$ is placed perpendicular to a uniform magnetic field $\vec{B}$ Two identical small balls of mass $m$and charge q each are attached at the rod ends. Let us direct z axis along the magnetic field and place the origin at the rod center. The balls are given the same initial velocity v_o but in opposite directions so that one of the velocities is precisely in the z-direction. What are the maximum coordinates $z_{max}$of the balls? Express your answer in terms of q, B, m, v and R Find the magnitude of the ball accelerations at this moment and express your answer in terms of q, B, m, v, R and z_{max}.

Referências

- Problema 1: Morin, Cap. 9, Problema 31
- Problema 2: A Guide 1, Cap. 1, Problema 39
- Problema 3: A Guide 1, Cap. 1, Problema 40
- Problema 4: Morin, Cap. 9, Problema 54
- Problema 5: 200 Puzzling 2, Problema 54
- Problema 6: 200 Puzzling 2, Problema 56
- Problema 7: Morin, Cap. 9, Problema 27
- Problema 8: Morin, Cap. 9, Problema 28
- Problema 9: Morin, Cap 9, Problema 29
- Problema 10: Morin, Cap 9, Problema 30
- Problema 11: Morin, Cap 9, Problema 56
- Problema 12: 200 Puzzling 2, Problema 57
- Problema 13: Inspirada na seção de nutação do Cap. 9 do Morin. 9.7.7 e exercício que tem em seguida.
- Problema 14: Apho 2017, Problema 3
- Problema 15: Ipho 2020, Problema 1, Item c